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Journal of Financial Intermediation 14 (2005) 351–375

Journal of Financial Intermediation

www.elsevier.com/locate/jfi

Relative performance objectives in financial markets ☆

Frederic Palomino

Department of Finance and Economics, HEC Paris, 78351 Jouy-en-Josas cedex, France

Received 1 November 2002

Available online 17 May 2005

Abstract

Money management is an activity in which agents are often evaluated on the basis of their relative performance. In this article, I consider an economy in which (i) investors use a relative performance rule to evaluate mutual fund managers and allocate money into funds, and (ii) fund managers receive an asset-based compensation. Such fund-picking rules and compensation schemes generate relative performance objectives for fund managers. I study the consequences of this for the mutual fund industry in terms of the number of competing funds and trading strategies. I show that, with respect to absolute performance maximization, relative performance objectives increase the riskiness of investment strategies and reduces the number of low-quality funds. Overall, relative performance objectives increase investors' expected return.

JEL classification: G14; G20

Keywords: Relative performance; Absolute performance

☆ A former version of this paper was circulated under the title "Relative Performance Equilibrium in Financial Markets."

E-mail address: palomino@hec.fr.

1042-9573/$ – see front matter
doi:10.1016/j.jfi.2004.09.004

1. Introduction

Money management falls into the category of activities in which agents are evaluated according to their relative performances. As observed by Lakonishok et al. (1992),

"*The objective difficulty in evaluating money managers' performance and separating 'luck' from 'skill' create agency problems between institutional money managers and fund sponsors. Typically, money managers are evaluated against each other*."

As a matter of fact, there is now extensive empirical evidence suggesting that investors allocate money into funds according to their relative past performance (see Ippolito, 1992; Sirri and Tuffano, 1998; Gruber, 1996; Chevalier and Ellison, 1997 and Lettau, 1997). Furthermore, open-end mutual fund managers often have asset-based compensation schemes (see Khorana, 1996 and Deli, 2002).[1] A direct consequence of the combination of fund managers' compensation schemes and investors' fund picking rule is that managers have relative performance objectives.

It is apparent that anticipation of being rewarded based on relative performance can affect both the entry decision and the investment decision of a fund conditional upon entry. For example, since relative performance provides greater rewards on average to managers with greater skills, the entry decision may depend on how one believes one's skills stack up against one's peers. Similarly, relative performance can affect a manager's investment decision via a propensity to alter the risk profile to win the "relative performance tournament." This raises the question I address in this paper: How are the entry and investment decisions of fund managers affected by relative performance objectives? In addressing this question, I consider an economy with risky assets traded in single-auction markets (as described in Kyle, 1985), and two types of funds managers, those with a low level of information and those with a high level of information.

Measuring the relative performance of a fund as the difference between its own profit and the average profit realized by its competitors, I show that relative performance objectives influence fund managers' behavior in two ways with respect to absolute profit maximization. First, the riskiness of investment strategies increases, i.e., for a given level of information, fund managers choose strategies that are riskier (in terms of variance of profits) relative to those aiming at maximizing absolute profits. Second, incentives to enter the industry are reduced for low-quality funds. The aggregate effect in an increase of funds' average performance. This suggests that investors are justified in using a relative-performance-based fund-picking strategy. Such a strategy increases the ratio of high-quality funds to low-quality funds, thereby increasing the probability that investors will put their money in high-quality funds.

As was shown in several papers, relative performance maximization in oligopolistic markets provides incentives to trade (or produce) larger quantities than those aiming at

[1] Khorana (1996, p. 406) mentions that "The investment advisor's compensation is directly linked to the fund's size; the advisor receives a management fee based on a percentage of average net assets held during the year." In a sample of 5198 funds, Deli (2002) finds that 4833 funds had advisory contracts based solely on a percent of assets.

maximizing absolute performance (see Fershtman and Judd, 1987; Salas-Fumas, 1992; Vega-Redondo, 1997 and Aggarwal and Samwick, 1999, in the case of standard Cournot markets). In financial markets, this means that agents use their information more intensively than absolute profit maximization suggests. A direct consequence is that more information is revealed by price. This implies that the variance of profit per unit traded decreases. My results show that the increase in the variance of profit due to the more intensive use of private information (i.e., trading of larger quantities) always dominates the decrease of the variance of profit due to higher informational efficiency. Hence, relative performance objectives generate investment strategies riskier than those aiming at maximizing absolute performance.

The intuition for the result about reduced entry is the following. If managers maximize their relative performance, their incentives to acquire information are affected in two ways. First, there is a "trading intensity" effect which reduces incentives to acquire information: managers know that if they buy information, they will trade very intensively, thereby realizing low profits. This effect illustrates the standard IO result that increased competition reduces profits, and thus ex ante incentives to invest in market participation. Second, there is a "difference-in-payoff" effect. It captures the fact that the lowest-quality funds always perform below average. As a consequence, the stronger the relative performance component in investors' strategy, the higher the cost (in terms of loss of inflows) paid by low-quality managers. As a consequence, their incentives to enter the industry decrease.

The organization of this paper is as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 derives the equilibrium and studies properties of investment strategies. Section 5 endogenizes the number of competing funds. Section 6 discusses the robustness of the results and Section 7 concludes.

2. Related literature

A growing body of literature studies the mutual fund tournament both theoretically and empirically. On the empirical side, Brown et al. (1996) and Chevalier and Ellison (1997) provide evidence that the mutual fund tournament generates incentives for managers not to act in the interest of investors. Assuming that managers are evaluated on a yearly basis, Brown et al. provide evidence that mid-year "losers" increase fund volatility in the latter part of the year to a greater extent than mid-year "winners." Chevalier and Ellison study how relative performance after three quarters of the year influence investment strategy in the last quarter. Considering two-year-old funds, they show that funds that are somewhat behind the market increase risk to a greater extent than funds that are ahead of the market.

Lynch Koski and Pontiff (1998) provide further evidence that change in risk is negatively correlated with prior performance within one calendar year. Goriaev et al. (2000) show that fund managers respond to ranking-based objectives, i.e., risk taking incentives increase with the distance to the top interim performer. Using a continuous-time approach, Chen and Pennacchi (1999) show that fund managers respond continuously to their relative performance. My results about trading strategies are consistent with these empirical results about fund managers' behavior.

Closely related theoretical papers studying relative performance evaluation in financial markets are those of Maug and Naik (1995), Gumbel (2004), Huddart (1999) and Taylor (2003). However, none of these models studies the impact of relative performance objectives on the degree of competition (i.e., the number of competing funds). Maug and Naik consider a situation in which a risk-averse manager is evaluated with respect to another trader. It is shown that relative performance objectives generate incentives to herd.

Gumbel (2004) considers a model of delegated portfolio management with two risk-neutral investors (principals) each of them employing one manager (agent). Principals decide *both* wage contracts and what asset their respective portfolio managers should trade. It is shown that principals propose relative performance contracts and herd on the same asset.

Huddart (1999) studies a two-period model with one informed and one uninformed fund. At the end of the first period, risk-averse investors (willing to invest in the informed fund) reallocate their wealth between two funds after having observed the portfolio and the performance of each fund. It is shown that asset-based compensation schemes generate incentives for managers to invest in overly-risky portfolios.

Finally, Taylor (2003) studies a two-period tournament between two fund managers with unequal mid-year performances competing to be the top performer at the end of the year. Taylor shows that the interim winner may gamble in the second part of the year as a best reply to the interim loser's "gambling-for-resurrection" strategy. The main difference between Taylor's model and mine is that he assumes that neither of the two competing managers possesses superior information or the possibility to acquire such information. Differences in incentives are just generated by exogenously-specified interim performances.

Grinblatt and Titman (1989a), Admati and Pfleiderer (1996) and Chen and Pennacchi (1999) study the consequences of a benchmark-adjusted compensation schemes. The general conclusion is that benchmark-adjusted compensation contracts are not in the interest of fund investors. The main difference between these models and mine comes from the choice of the benchmark. They assume that a fund manager faces no competition and thus is evaluated with respect to an *exogenous* benchmark portfolio. To motivate this choice, Admati and Pfleiderer give the following example. "Suppose a portfolio manager investing in Japanese stocks realizes a return of 27%. How well has she performed? A good answer to this question cannot be given without knowing how well Japanese stocks performed in the aggregate." The question posed here is different: in an economy with several competing fund managers, if one of them has outperformed a prespecified benchmark portfolio by 2%, how well has she performed? The answer is that this probably depends on the performance of her competitors. Therefore, I study the influence of an *endogenous* benchmark (which represents the performance of all competing funds) on the portfolio decision of each fund manager.

Finally, the literature on the evaluation of agents in groups (see, for example, Lazear and Rosen, 1981; Holmström, 1982; Green and Stockey, 1983 and Mookherjee, 1984) derives conditions on agents' realized output under which compensation based relative performance is preferable to compensation based on individual performance. Roughly, relative-performance-based compensation should be used if each agents' output depends on effort, idiosyncratic noise, a common shock experienced by all agents and a common risk which is large relative to the idiosyncratic risk. In this paper, I do not question whether allocating capital into funds according to past relative performances is optimal behavior on the part of fund investors. Rather, I study some of the *consequences* of such behavior. Furthermore, in my model, portfolio decisions are costless, i.e., they do not require any effort from fund managers.

3. The model

There are N funds that initially have the same amount of asset under management W_0. Funds are split into two categories: low-quality funds and high-quality funds.

There are two periods: 1 and 2. In period 1, the economy contains three assets: a risk-free bond available in infinitely elastic supply with return normalized to one, and two risky assets ($i = 1, 2$). Risky assets are traded in single-auction markets similar to those described by Kyle (1985). After trade occurs, the liquidation value $\tilde{v}_i$ of asset i is realized.

There are $2N_l > 0$ low-quality funds and $N_h > 0$ high-quality funds. Low-quality funds are split into two groups. N_l fund managers are informed about asset 1 and N_l are informed about asset 2. We denote $G_{l,i}$ ($i = 1, 2$) the group of low-quality managers with information about asset i. In each group, funds are identical. At the beginning of period 1, fund managers of group $G_{l,i}$ observe a signal $\tilde{s}_i = \tilde{v}_i + \tilde{e}_i$.

All high-quality funds are identical. They have private information about *both* assets 1 and 2. At the beginning of period 1, high-quality fund managers observe both signals $\tilde{s}_1$ and $\tilde{s}_2$.

There are three types of market participants: funds, noise traders and market makers. In each market, noise traders together submit an exogenous random quantity $\tilde{z}_i$. The market maker observes the aggregate demand, clears the market and sets prices $\tilde{p}_i$ efficiently (in the semi-strong form) conditional on market aggregate demands. It is assumed that $\tilde{v}_i$, $\tilde{e}_i$, and $\tilde{z}_i$ are independently normally distributed with mean zero and variance σ_v^2, τ^{-1}, and σ_z^2, respectively.

Given these assumptions about the distribution of uncertainty, all low-quality managers have information of the same precision. The only difference between low-quality managers and high-quality managers is that managers of the latter type have information about two assets while managers of the earlier type have information about only one asset. Moreover, the degrees of competitiveness of markets 1 and 2 are the same since they have the same number of agents with information of the same precision.

Fund inflows. Let $N = N_h + 2N_l$ and denote π_n the profit realized by fund n ($n = 1, \ldots, N$) in period 1. At the end of period 1, investors observe the profit realized by each fund n in period 1 (i.e., π_n). At the beginning of period 2, investors allocate money into fund n according to its relative performance in period 1. The amount, K_n, allocated to fund n is given by:

$$K_n = k\left(\pi_n - \frac{a}{N}\sum_{j=1}^{N}\pi_j\right) \tag{1}$$

with $k > 0$ and $a \in [0, 1]$.

The coefficient a measures the intensity with which investors use a relative performance rule to allocate money into funds. If $a = 0$, investors allocate money into funds independently of the performance of their direct competitors. If $a > 0$, then the effect of the performance of competitors on the amount invested in fund n is increasing in a.

It should be remarked that π_n is included in the benchmark against which manager n is evaluated. The analysis is not qualitatively modified if, instead, it is assumed that the benchmark against which a fund is evaluated is the average performance realized by its $(N - 1)$ competitors. To see this, rewrite Eq. (1) as

$$K_n = k'\left(\pi_n - \frac{a'}{N}\sum_{j=1, j\neq n}^{N} \pi_j\right) \tag{2}$$

with

$$k' = k\frac{N-a}{N} \quad \text{and} \quad a' = \frac{a}{N-a}. \tag{3}$$

Given that a' is increasing in a, the analyses with Eqs. (1) and (2) are qualitatively equivalent. More precisely, all comparative static results with respect to the relative performance coefficient (i.e., sign of the derivatives) obtained under specification (1) also hold under specification (2).

Managers' compensation. Managers are compensated in period 2 after allocation of money into funds by investors. Managers receive a compensation proportional to the size of assets under management at that moment:

$$C_n = cW_{2,n}$$

where $W_{2,n}$ represents the size of assets under management at the beginning of period 2 for manager n.

Given investors' allocation rule, we can rewrite C_n as

$$C_n = c[W_0 + \pi_n + K_n]. \tag{4}$$

The first two terms $(W_0 + \pi_n)$ represent the size of assets under management at the end of period 1.

Putting (1) into (4), we obtain

$$C_n = c\left\{W_0 + (1+k)\left(\pi_n - \frac{\alpha}{N}\sum_{j=1}^{N}\pi_j\right)\right\} \tag{5}$$

where

$$\alpha = \frac{ak}{1+k}. \tag{6}$$

Since α is proportional to a, α also measures the intensity with which investors use a relative performance rule to allocate money into funds.

The model captures the following ideas in a simple framework:

(i) In each category of funds,[2] there are good and bad fund managers and this difference in quality is persistent over time[3];
(ii) For a given category, investors would like to invest in funds managed by good managers and use relative performance as a rule of thumb to evaluate managers and allocate capital into funds[4];
(iii) Fund managers are risk-neutral agents who are compensated on the basis of the size of the fund they manage.[5]

Under such circumstances, each manager's objective is to maximize the size of her fund. Given the way investors allocate money into funds and managers' compensation schemes, the objective function of the manager in period 1 has a relative performance component.

Before proceeding, several remarks should be made. First, it is assumed that portfolios are unobservable. I believe the assumption is realistic since portfolio disclosure is not frequent[6] and managers window-dress their portfolio around disclosure dates in practice.[7]

Second, it is assumed that funds have market power. This assumption follows from evidence that trades by some institutional investors influence prices.[8] Hence, I think that our model is better suited than one of perfect competition to explain the behavior of funds specialized in small-cap stocks.

Also, the number of funds competing in a given category is not always very large. Gould (1998) illustrates this when writing about one specific fund: "Bartlett Europe has returned an annual average of 27.2 percent for the three years through Dec. 4, ranking first among the 46 European stock funds tracked by Morningstar Inc., the financial publisher. The average return of the category was 18.7 percent, Morningstar said." Given that the Morningstar ranking is highly regarded by investors, it is likely to be the case that each of these 46 funds mainly competes against the 45 others.

4. Equilibrium

Given the assumptions about the distribution of the uncertainty in the economy, managers' investment strategies in each market can be studied separately. Denote $x_{n,i}$ the amount invested by manager n ($n = 1, \ldots, N$) in market i ($i = 1, 2$). Let $x_n = (x_{n,1}, x_{n,2})$, $X_{-\{n\},i} = (x_{1,i}, \ldots, x_{n-1,i}, x_{n+1,i}, \ldots, x_{N,i})$, and $X_{-\{n\}} = (x_1, \ldots, x_{n-1}, x_{n+1}, \ldots, x_N)$.

2 By category of funds, I mean funds with the same investment objective. The Investment Company Institute classifies US Mutual Funds in 33 investment categories (see (Investment Company Institute, 2003)).

3 See Grinblatt and Titman (1992), Hendricks et al. (1993), Brown and Goetzman (1995) and Carhart (1997). Furthermore, evidence of persistent underperformance seems to be stronger than that of over-performance.

4 See Ippolito (1992), Sirri and Tuffano (1998), Chevalier and Ellison (1997) and Lettau (1997) for empirical evidence of such a behavior.

5 See Deli (2002) for evidence of such compensation schemes.

6 In the United States, mutual fund portfolios have to be disclosed semiannually. However, other countries such as the Netherlands require disclosure only once a year.

7 See Lakonishok et al. (1992), Musto (1999) and Carhart et al. (2002).

8 See Grinblatt and Titman (1989b) and Lakonishok et al. (1992) and the Vanguard Group admitting that their small-cap index funds affect small-cap stock prices (see Wyatt, 1998).

Denote $\pi(x_{n,i}, X_{-\{n\},i})$, the profit realized in market i by manager n as a function of the quantity he trades ($x_{n,i}$) and the vector of quantities traded by his competitors ($X_{-\{n\},i}$).[9]

The compensation of manager n as a function of his investment strategy is then given by

$$C_n(x_n, X_{-\{n\}}) = c\left\{W_0 + (1+k)\sum_{i=1}^{2}\left(\pi(x_{n,i}, X_{-\{n\},i}) - \frac{\alpha}{N}\sum_{j=1}^{N}\pi(x_{j,i}, X_{-\{j\},i})\right)\right\}. \tag{7}$$

Let I_n denote the information of manager n. (Hence, if manager n belongs to $G_{l,i}$ ($i = 1, 2$) then $I_n = \{s_i\}$ while if manager n is of high quality then $I_n = \{s_1, s_2\}$.) Given the market game I consider, an equilibrium is defined as follows.

Definition. An equilibrium is a pair of price vectors $p^* = (\tilde{p}_1^*, \tilde{p}_2^*)$ and vectors $X^* = (x_1^*(I_1), \ldots, x_N^*(I_N))$ such that for all $i = 1, 2$,

$$\tilde{p}_i^* = E\left(\tilde{v}_i \,\middle|\, \sum_{n=1}^{N}\tilde{x}_{n,i}^* + \tilde{z}_i\right) \tag{8}$$

and for all $n = 1, \ldots, N$,

$$x_n^* \in \operatorname{Argmax}_x E\big[C_n\big(x, X_{-\{n\}}^*\big) \mid I_n\big]. \tag{9}$$

Condition (8) states that prices are efficient (in the semi-strong form) in equilibrium while condition (9) states that managers maximizes their expected compensation.

Proposition 1. *There exists a unique symmetric linear equilibrium. It has the following properties. Let $M = N_h + N_l$ and*

$$\beta^*(\alpha) = \frac{\sigma_z\sqrt{(N-\alpha)\tau}}{\sqrt{M(N-\alpha M)(\sigma_v^2\tau + 1)}},$$
$$\lambda^*(\alpha) = \frac{\sigma_v^2\sqrt{M(N-\alpha M)(N-\alpha)\tau}}{\sigma_z[2(N-\alpha) + (M-1)(N-2\alpha)]\sqrt{(\sigma_v^2\tau + 1)}}.$$

(i) *Low-quality managers from $G_{l,1}$ and $G_{l,2}$ trade vectors of quantities $(\beta^*(\alpha)\tilde{s}_1, 0)$ and $(0, \beta^*(\alpha)\tilde{s}_2)$, respectively.*
(ii) *High-quality managers trade vectors of quantities $(\beta^*(\alpha)\tilde{s}_1, \beta^*(\alpha)\tilde{s}_2)$.*
(iii) $\tilde{p}_i^* = \lambda^*(\alpha)(M\beta^*(\alpha)\tilde{s}_i + \tilde{z}_i)$.

Proof. See Appendix A. □

[9] The period-1 realized profit, π_n (see paragraph *Fund inflows* in the previous section), is then

$$\pi_n = \pi(x_{n,1}, X_{-\{n\},1}) + \pi(x_{n,2}, X_{-\{n\},2}).$$

The proposition derives the equilibrium prices and investment strategies as functions of the relative performance component α. Also, low-quality managers from group G_{li} only trade asset i while high-quality managers trade both assets. Hence, M represents the number of agents effectively trading in each market.

Since I am interested in the influence of the relative performance objectives on investment strategies, I want to study the impact of α on trading strategies. The following results are derived.

Proposition 2. (i) *Trading aggressiveness is increasing in* α, *i.e.*, $\partial\beta^*(\alpha)/\partial\alpha > 0$.
(ii) *The ex ante variance of profit is increasing in* α, *i.e.*, $\partial \operatorname{Var}(\pi_n^*)/\partial\alpha > 0$.

Proof. See Appendix A. □

Part (i) of the proposition states that relative performance objectives generate incentives for managers to overinvest, i.e., to trade larger quantities than those aiming at maximizing absolute profits. This result is similar to those obtained by Fershtman and Judd (1987), Salas-Fumas (1992), Vega-Redondo (1997) and Aggarwal and Samwick (1999) in the case of standard Cournot markets: as long as the price is larger than the marginal cost, a firm must sell a larger quantity than its competitors in order to outperform them. Hence, at the Cournot–Nash equilibrium quantity, firms have incentives to deviate and produce a larger quantity.

In financial markets, the intuition is the same. The presence of noise traders implies that the marginal profit is always positive, i.e., $|E(v_i \mid s_i) - p_i| > 0$. Now, without loss of generality, assume that $s_i > 0$. In such a case, all managers trade positive quantities. Assume that they all trade the same quantity $x > 0$. If a manager with relative performance objectives (i.e., $\alpha > 0$) deviates from x and trades a quantity $x' > x$, her compensation is affected in two ways. First, she decreases her compensation via a decrease in her absolute profit, and second, increases her compensation by outperforming her competitors given that all managers trade at the same price. These two effects imply that if $\alpha > 0$, then in equilibrium the marginal absolute profit is negative.

Note that we always have $\alpha < 1$ given that $\alpha = ak/(1+k)$ and $a \leqslant 1$. In the case in which we have $\alpha = 1$, then an equilibrium fails to exist if $N_h = N$. The reason is that in this case

$$K_n + \pi_n = \pi_n - \frac{1}{N}\sum_j^N \pi_j.$$

Given that the marginal profit ($|E(v_i \mid s_i) - p_i|$) is always positive, expected profits are always positive. Therefore, a necessary condition for a manager's compensation to be maximized is that this manager trades a quantity larger than the average trading quantity. Hence, all managers want to be above average in terms of trading intensity. It follows that an equilibrium fails to exist.

In the general case ($\alpha \in (0,1)$), the equilibrium trading aggressiveness influences the riskiness of the strategy (i.e., the ex-ante variance of profits) in two ways. First, the variance is increased through trading of larger quantities. Second, the variance is decreased through a higher level of informational efficiency (i.e., the more aggressive the trading strategy, the greater the amount of information is revealed by price (see Kyle, 1985).

Part (ii) of the proposition states that the first effect always dominates the second and relative performance objectives increase the riskiness of investment strategies—increases the ex-ante variance of profits—with respect to absolute performance objectives. The reason is the following. The profit realized in market i can be written as $\beta(v_i + e_i)[v_i - \lambda(M_i\beta(v_i + e_i) + z_i)]$. Therefore, the profit variance can be decomposed into three components: the fundamental risk (related to the fundamental value of the asset, i.e., v_i), the noisy-signal component (i.e., related to e_i) and the noise-trading component (related to z_i). As α increases, fund managers increase their trading intensity (β). In response, λ increases. Hence, the product $\beta\lambda$ increases, which in turn increases the noisy-signal and noise-trading components of the variance. Finally, as shown in the proof of Proposition 2, $\beta(1 - M\lambda\beta)$ is also an increasing function of α. This means that the reduction of fundamental risk due to an increase of information revelation is dominated by the increase of fundamental risk due to increased trading aggressiveness. The reason is that the market maker cannot disentangle increased demand coming from fund managers from noise trading. As a consequence, the reduction of fundamental risk due to information revelation is small relative to the increased trading aggressiveness, so the fundamental risk component of the variance also increases. Hence, the three components of the variance increase as α increases.

These results should be compared to those of Brown et al. (1996) and Chevalier and Ellison (1997). Both papers assume that managers make several investment decisions during the evaluation period and act as price takers. They provide evidence that information about intermediate performance generates incentives to increase the riskiness of investment strategy. However, in their framework, if managers were evaluated after each investment decision, then these incentives would disappear. Proposition 2 establishes that if managers have market power, then relative performance objectives increase the riskiness of investment strategies even if managers are evaluated after each investment decision.

5. Endogenous number of competing funds

In this section, I investigate how relative performance objectives influence the size of the mutual fund industry, i.e., the number of competing funds. To do this, I modify slightly the model of the previous section as follows. I assume that there are $\overline{N}_h > 1$ high-quality managers and an infinite number of low-quality managers. Low-quality managers are split into two groups (G_{l1} and G_{l2}). All managers have a reservation compensation R. At the beginning of period 1, low-quality managers from Group G_{li} ($i = 1, 2$) can acquire information about asset i (i.e., observe s_i) at an effort cost Q. High-quality managers can acquire information about *both* assets 1 and 2 at the effort cost Q. If a manager decides to enter the industry and open a fund, she receives an amount to manage W_0 from investors. It will be assumed that W_0 is small so that $cW_0 < R$.

5.1. Equilibrium

Using the result of Proposition 1, one deduces that only funds informed about asset i ($i = 1, 2$) trade asset i. Therefore, the inequality $cW_0 < R$ implies that only funds that have exerted effort enter the industry and trade. Uninformed managers have an expected compensation smaller than their reservation compensation, and are therefore better off staying out.

Assume that $N_h \leqslant \overline{N}_h$ high-quality managers and N_{li} low-quality managers acquire information about asset i. Then, the number of funds trading asset i is $M_i = N_h + N_{li}$ and the total number of competing funds is $N = N_h + N_{l1} + N_{l2} = M_1 + N_{l2} = M_2 + N_{l1}$. From Proposition 1, one deduces that the expected profit from trading a quantity $\beta(\alpha)\tilde{s}_i$ in market i if there are M_i funds trading asset i is

$$\Pi(M_i, \alpha) = \frac{\sigma_z \sigma_v^2 \sqrt{(N - \alpha M_i)(N - \alpha)\tau}}{[2(N - \alpha) + (M_i - 1)(N - 2\alpha)]\sqrt{M_i(\sigma_v^2 \tau + 1)}}. \tag{10}$$

Hence, if N_{li} ($i = 1, 2$) low-quality funds from G_{li} enter the industry, then the expected compensation (net of effort cost) of a high-quality manager acquiring information if there are N_h informed high-quality managers is

$$\begin{aligned} &F_h(N_h, N_{l1}, N_{l2}, \alpha) \\ &\quad = c(1+k) \sum_{i=1,2} \left\{ \left(1 - \frac{\alpha(N_h + N_{li})}{N_h + N_{l1} + N_{l2}}\right) \Pi(N_h + N_{li}, \alpha) \right\} + cW_0 - Q. \end{aligned}$$

The expected compensation of a low-quality manager from G_{li} who acquires information about asset i if there are N_{li} low-quality managers informed about asset i, N_{lj} low-quality managers informed about asset j ($j = 1, 2$, $j \neq i$), and N_h informed high-quality managers, is

$$\begin{aligned} F_l(N_h, N_{li}, N_{lj}, \alpha) = +c(1+k) \Bigg\{ &\left(1 - \frac{\alpha(N_h + N_{li})}{N_h + N_{l1} + N_{l2}}\right) \Pi(N_h + N_{li}, \alpha) \\ &- \frac{\alpha(N_h + N_{lj})}{N_h + N_{l1} + N_{l2}} \Pi(N_h + N_{lj}, \alpha) \Bigg\} + cW_0 - Q. \end{aligned}$$

The equilibrium numbers of competing funds $(N_h^*, N_{l1}^*, N_{l2}^*)$ satisfy the following conditions:

$$F_h\big(N_h^*, N_{l1}^*, N_{l2}^*, \alpha\big) > R > F_h\big(N_h^* + 1, N_{l1}^*, N_{l2}^*, \alpha\big), \tag{11}$$

$$F_l\big(N_h^*, N_{li}^*, N_{lj}^*, \alpha\big) > R > F_l\big(N_h^*, N_{li}^* + 1, N_{lj}^*, \alpha\big), \tag{12}$$

with $i = 1, 2$ and $i \neq j$.

If only high-quality managers enter the industry, then their expected compensation is

$$F_h(N_h, 0, 0, \alpha) = c\big[W_0 + 2(1+k)(1-\alpha)\Pi(N_h, \alpha)\big] - Q.$$

Simple derivative calculation shows that $F_h(N_h, 0, 0, \alpha)$ is a decreasing function of α. Furthermore, for any $N_h > 1$, $\mathrm{Lim}_{\alpha \to 1} F_h(N_h, 0, 0, \alpha) = cW_0 - Q$. As a consequence, if α is large, i.e., close to 1, $F_h(N_h, 0, 0, \alpha) < R$ and only one high-quality fund enters the industry.[10] Applying the same reasoning for low-quality funds, I obtain that if α is large and $N_h = 1$, then $N_{l1} = N_{l2} = 0$ since $F_l(1, 1, 0, \alpha) < F_h(2, 0, 0, \alpha) < R$. As a consequence, the mutual fund industry is restricted to one single fund.

To avoid this single-fund-industry case, it is assumed that $\overline{N}_h$, Q, and R are such that there exists $\bar{\alpha} > 0$ such that $F_h(\overline{N}_h, 0, 0, \bar{\alpha}) = R$. Furthermore, to keep the model tractable, I assume that N_h and N_{li} $(i = 1, 2)$ are real numbers.

Lemma 3. *Assume N_h, N_{li} $(i = 1, 2)$ are real numbers, and $(N_h^*, N_{l1}^*, N_{l2}^*)$ is an equilibrium. If $N_h^* < \overline{N}_h$, then $N_{li}^* = 0$ $(i = 1, 2)$ and if $N_h^* = \overline{N}_h$, then $N_{li}^* \geqslant 0$ $(i = 1, 2)$.*

Proof. See Appendix A. □

Lemma 3 implies that a necessary condition to have low-quality and high-quality funds both entering the industry is that $\alpha < \bar{\alpha}$, so that $F_h(\overline{N}_h, 0, 0, \alpha) > R$. In such a case, $N_h^* = \overline{N}_h$ and $N_{li}^* \geqslant 0$ $(i = 1, 2)$ in equilibrium. In what follows, I concentrate on such equilibria that I call *two-type* equilibria.

Lemma 4. *Assume that N_{li} $(i = 1, 2)$ is a real number. For any N_{l1} and N_{l2},*

$$\frac{\partial F_l}{\partial N_{l1}}\big(\overline{N}_h, N_{l1}, N_{l2}, \alpha\big) < 0 \quad \textit{and} \quad \frac{\partial F_l}{\partial N_{l2}}\big(\overline{N}_h, N_{l1}, N_{l2}, \alpha\big) > 0.$$

Proof. See Appendix A. □

Lemma 4 implies that any two-type equilibrium is such that $N_{l1}^* = N_{l2}^*$. Denote $N_{li}^*(\alpha)$ the equilibrium number of low-quality funds from Group G_{li} entering the industry as a function of α. The following results are derived.

Proposition 5. *Assume that N_{li} $(i = 1, 2)$ is a real number. For any set of parameters such that $F_h(\overline{N}_h, 0, 0, \alpha) > R$,*

(i) *There exists $\bar{\bar{\alpha}} > 0$ such that for any $\alpha < \bar{\bar{\alpha}}$, there exists a* unique *two-type equilibrium.*
(ii) *If $N_{li}^*(0) > 0$, then for any α' and α'' such that $0 < \alpha' < \alpha'' < \bar{\bar{\alpha}}$, $N_{li}^*(0) > N_{li}^*(\alpha') \geqslant N_{li}^*(\alpha'')$.*

Proof. See Appendix A. □

Part (i) of the proposition establishes that there exists a threshold $\bar{\bar{\alpha}} > 0$ below which there is a unique two-type equilibrium. This implies that on the interval $[0, \bar{\bar{\alpha}}]$, comparative static can be performed. The first result (part (ii) of the proposition) is that relative performance objectives reduce the number of low-quality funds in the industry. Furthermore, if $N_{li}^*(\alpha') > 0$, then the strict inequality $N_{li}^*(\alpha') > N_{li}^*(\alpha'')$ holds.

[10] Given that the entering fund realizes the monopoly profit, entering is profitable for this fund.

The reason for the reduced entry of low-quality funds is the following. Assume that investors allocate money into funds according to their absolute performance. Then, the expected compensation (net of effort cost) of a low-quality manager is $c(1+k)\Pi(\overline{M},0)+cW_0-Q$. If, in such a case, there are $\overline{M}$ funds trading in a given market, then $c(1+k)\Pi(\overline{M},0)+cW_0-Q=R$. Relative performance objectives modify incentives to exert effort and enter the industry in two ways. First, there is a *trading intensity effect* as established by Proposition 2: $\beta(\alpha)>\beta(0)$ which implies that $\Pi(\overline{M},\alpha)<\Pi(\overline{M},0)$. This effect reduces incentives to enter the industry.

The second effect I call *difference-in-payoff effect*. It captures how relative performance objectives modify the expected compensation, independently of the trading intensity. To isolate this effect, consider a situation in which managers take into account their relative performance objective when buying information but behave as absolute profit maximizers when trading (i.e., when choosing β). If I denote $\widehat{\Pi}(M,\alpha)$, the expected profit in this situation as a function of M and α, then for all $\alpha\in(0,1)$, $\widehat{\Pi}(M,\alpha)=\widehat{\Pi}(M,0)=\Pi(M,0)$.

Given that there are two categories of funds in the industry, low-quality funds always perform below average. As a consequence, the stronger the relative performance component in investors' strategy (i.e., the larger α), the stronger the cost (the loss of compensation due to an outflow) incurred by low-quality managers. As a consequence, their incentives to enter the industry decrease. Hence, the larger α, the smaller the number of low-quality funds entering the industry.

The difference-in-payoff effect also shows that the reduction in the number of competing funds is not (only) due to the traditional IO result that increased competition reduces profit and therefore the incentive to invest in market presence. Even if trading strategies were not affected by the relative performance component, relative performance evaluation would decrease the number of competing funds compared to the number with absolute performance evaluation.

5.2. Industry performance

Now I turn to the impact of relative performance evaluation on the expected return investors obtain from investing in funds. Given that there are several categories of funds, I measure the overall performance of the industry by the average expected profit of the N competing funds ($\overline{\Pi}(\alpha)$). Let $N_l^*(\alpha)=N_{li}^*(\alpha)$ $(i=1,2)$.

$$\overline{\Pi}(\alpha)=\frac{2\overline{N}_h+2N_l^*(\alpha)}{\overline{N}_h+2N_l^*(\alpha)}\Pi\big(\overline{N}_h+N_l^*(\alpha),\alpha\big).$$

Proposition 6. *Assume that N_l is a real number. For any α' and α'' such that $\alpha'<\alpha''<\bar{\bar{\alpha}}$, $\overline{\Pi}(\alpha')<\overline{\Pi}(\alpha'')$.*

Proof. See Appendix A. □

Proposition 6 establishes that the stronger the relative performance component in investors' fund-picking strategy, the higher the expected performance of the industry. The reason is that when α increases, three effects are at work. First, the trading intensity and difference-in-payoff effects described above and, second, the ratio of high-quality funds relative to low-quality fund increases. The trading intensity effect decreases absolute profits, hence the average profit. The difference-in-payoff effect increases absolute profits through a decrease of the number of market participants, and thus increases the average profit. Finally, the increase of the ratio of high-quality to low-quality funds increases the average payoff. The proposition states that the two positive effects dominate the negative one.

This result suggests that investors are justified in using a relative performance component when choosing among funds. Relative-performance-based fund-picking strategies reduce low-quality managers' incentive to enter the industry (since they will perform below average). Hence, investors' probability of investing in a high-quality funds increases, and so does their expected return, even though relative performance also increases the trading aggressiveness of the competing funds.

It is worth mentioning that the same result holds if only high-quality funds enter the industry.[11] Assuming N_h continuous, N_h^* will be such that $F(N_h^*, 0, 0, \alpha) = R$. This is equivalent to

$$\Pi\left(N_h^*, \alpha\right) = \frac{2}{(1+k)(1-\alpha)} \frac{Q+R}{c}.$$

Hence, funds' expected profit is an increasing function of α.

5.3. Informational efficiency

Propositions 2 and 5 show that relative performance objectives influence both the number of competing funds and the utilization of the information they acquire. This influences the level of informational efficiency in two ways. First, managers trade more aggressively, thereby increasing informational efficiency. Second, the larger α, the smaller the equilibrium number of competing funds, which decreases informational efficiency. The following proposition states that, at least when α is small, the first effect always dominates the second one.

Proposition 7. *Assume that N_l^* is a real number. Then there exists $\hat{\alpha} \leqslant \bar{\bar{\alpha}}$ such that* $\mathrm{Var}(\tilde{v}_i \mid p_i)$ $(i = 1, 2)$ *is a strictly decreasing function of α on* $[0, \hat{\alpha}]$.

Proof. See Appendix A. □

5.4. Comment

The previous analysis assumes that N_h and N_l are real numbers. However, they are integer numbers. This means that in the case of two-type equilibria, for given α' and α'' $(\alpha' \neq \alpha'')$ and a small variation $\mathrm{d}\alpha$, we can have either $N_l^*(\alpha' + \mathrm{d}\alpha) = N^*(\alpha')$ or $N_l^*(\alpha'' + \mathrm{d}\alpha) = N_l^*(\alpha'') - 1$. In the case $N_l^*(\alpha' + \mathrm{d}\alpha) = N^*(\alpha')$, a marginal increase in α has an impact only on trading aggressiveness, which goes up. It implies that

[11] Remember, this happens if $N_h^* < \overline{N}$, a sufficient condition being $F(\overline{N}_h, 0, 0, \alpha) < R$.

$\overline{\Pi}(\alpha' + \mathrm{d}\alpha) < \overline{\Pi}(\alpha')$ and the level of informational efficiency increases. By contrast, in the case $N_l^*(\alpha'' + \mathrm{d}\alpha) = N_l^*(\alpha'') - 1$, a marginal increase in α has two effects. First, it has a marginal positive impact on trading aggressiveness. Second, it has a large negative effect on the number of low-quality funds entering the industry (i.e., $N_l^*(\alpha'' + \mathrm{d}\alpha) = N_l^*(\alpha'') - 1$), and thus a large positive impact of the average profit. The increase in the average profit is due to the facts that all profits increase through a decreased number of competing funds and the ratio of high-quality to low-quality funds also increases. There is also a strong negative impact on the level of informational efficiency since the number of agents participating in the markets is reduced.

With integer numbers then, Propositions 6 and 7 should be interpreted as follows. For any α' and α'' ($\alpha'' > \alpha'$) such that the solutions N_l' of $F(\overline{N}_h, N_l', N_l', \alpha') = R$ and N'' of $F(\overline{N}_h, N_l'', N_l'', \alpha'') = R$ are integer numbers, $\overline{\Pi}(\alpha') < \overline{\Pi}(\alpha'')$, and the level of informational efficiency is higher at α'' than at α'. That is, for variations of α large enough to generate a variation of the number of funds entering the industry, the average industry performance (i.e., the average profit) and the level of informational efficiency increase.

6. Robustness of the results

One may wonder how my results are influenced by some of the assumptions of the model, above all those about risk-neutrality, the structure of the private information and the linearity of the relative performance objectives.

6.1. *Linear relative performance objective*

An assumption of my model is that investors use a linear relative performance rule to pick funds; this rule generating linear relative performance objectives for fund managers. The rule investors use is exogenous and is chosen for tractability. A natural alternative fund-picking rule is one based on rankings. I will show, however, that if funds charge load fees, a rule identifying the best performer as the best-informed manager generates an objective function for fund managers relatively "close" to the linear relative performance objective function.

To see this, note that if funds charge load fees, then investors differentiate between money already invested in some funds and money to be invested for the first time.[12] In my model, given managers' strategy, the allocation rule investors should use is such that the money to be invested for the first time goes to the best-performing fund, and the money already invested either stays in the same fund or moves to the best-performing one, depending on the difference in expected profits generated by a low-performance manager and the expected profit net of load fees generated by the best-performing manager.

Given such a money allocation rule, what is the objective of a fund manager? If investors do not know the precision of managers' information, they do not know their expected profit.

[12] As empirical evidence shows, fund picking strategies are different for these two types of money. See, for example, Gruber (1996).

As a consequence, the worse the performance realized by a fund, the smaller the expected profit inferred by investors. It follows that for a manager, the lower his performance, the higher the probability that money invested in his fund will move to another fund. Therefore, the objective of a manager should have three components. First, the maximization of the probability of being ranked first, in order to maximize the probability of winning the money to be invested for the first time. Second, conditional on being ranked first, the maximization of the (positive) difference between his performance and the average performance realized by his opponents in order to maximize the probability of receiving money already invested in another fund. Third, conditional on not being ranked first, the minimization of the (negative) difference between his performance and the best performance, in order to minimize the probability of losing money already invested in his fund. These three components produce an objective function for manager such that the smaller the amount of money to be invested for the first time relative to the amount of money already invested, the "closer" the objective function of managers to the linear relative performance. Therefore, in the extreme case in which managers compete only for money already invested, their objective is close to the linear relative performance.

Hence, when the fund-picking rule used by investors is based on rankings, the behavior of manager should be "close" to that described in our model.

6.2. Risk-averse fund managers

My model assumes that fund managers are risk-neutral agents aiming at maximizing their expected compensation. One may wonder how risk neutrality influences their trading aggressiveness and their willingness to acquire information. This case is partially studied in Appendix A where it is shown that if managers have negative exponential utility functions, then results about trading aggressiveness still hold.

6.3. Structure of the private information

I have assumed that the difference between managers is that some of them are informed about the two assets while the others are informed about only one asset.

An alternative framework is with only one risky asset and the two types of managers having private signals of different precision. Under such an assumption, there exist sets of parameters such that the results about the riskiness of investment strategies and information acquisition hold. However, I cannot assert that this is true for any set of parameters.

7. Conclusion

Even in absence of relative-performance-based compensation schemes, the competition for funds between money managers may generate relative performance objectives.

In this article, I have analyzed fund managers' behavior when their objective function has a relative performance component. I have shown that relative performance objectives influence fund managers' behavior in two ways. First, relative performance objectives increase trading aggressiveness, i.e., managers use their private information more intensively than they would do if maximizing their absolute performance. Second, relative performance objectives reduce low-quality managers' incentives to enter the industry. The aggregate effect is an increase of the average performance in the industry.

These results suggest that investors may be justified in using a relative-performance based fund-picking strategy. Such a strategy increases the ratio of high-quality funds to low-quality funds, thereby increasing the probability that investors put their money in high-quality funds.

Acknowledgments

I thank Hans Degryse, Gul Gurkan, Martin Lettau, Aldo Rustichini, Joszef Sakovics, Harald Uhlig, and seminar participants at CEMFI, ESSEC, FMG Lunchtime Seminar (LSE), HEC, THEMA, UNSW, Sydney University, Paris-Dauphine and Pompeu Fabra for helpful comments. This project was initiated while I was visiting the University of Alicante. I would like to thank this institution for its hospitality. Financial support from the TMR network on "The Industrial Organization of Banking and Financial Markets in Europe" is acknowledged.

Appendix A

Proof of Proposition 1. First, note that given the assumptions about the distribution of uncertainty in the model, the relative performance of a manager is the sum of the performances realized in the two markets and the relative profit realized in a particular market is independent of the strategies played in the other markets. Second, it is straightforward that managers who do not have information about asset i ($i = 1, 2$) do not trade this asset since they expect a loss from trade (they have less information than market makers who observe the aggregate order flow). Third, if the M managers informed about asset i play a strategy $x_i(s_i) = \beta s_i$, then the market maker set a linear price $p_i = \lambda(M\beta s_i + z_i)$ where

$$\lambda = \frac{M\beta\sigma_v^2\tau}{M^2\beta^2(\sigma_v^2\tau + 1) + \sigma_z^2\tau}. \tag{A.1}$$

Now, assume that the market maker sets a linear price $p(y) = \lambda_i y_i$ where y_i represents the aggregate order flow in market i and $M - 1$ informed managers informed about asset i play the same linear strategy βs_i. Then, the expected compensation generated by trading in market i of the last manager informed about asset i if he trades a quantity x is

$$\begin{aligned} &E\big(C_i(x) \mid s_n\big) \\ &\quad = (1+k)\big(E(v_i \mid s_i) - \lambda\big[(M-1)\beta s_i + x\big]\big)\left(\frac{N-\alpha}{N}x - \frac{(M-1)\alpha}{N}\beta s_i\right). \end{aligned} \tag{A.2}$$

The first-order condition of expected-compensation maximization yields

$$x = \frac{(N-\alpha)E(v_i \mid s_i) - (M-1)(N-2\alpha)\lambda\beta s_i}{2(N-\alpha)\lambda}. \tag{A.3}$$

It follows that the manager plays a linear strategy $x = \beta' s_i$, where

$$\beta' = \frac{(N-\alpha)\sigma_v^2\tau - (N-2\alpha)(M-1)(\sigma_v^2\tau+1)\lambda\beta}{2(N-\alpha)(\sigma_v^2\tau+1)\lambda}. \tag{A.4}$$

In a symmetric equilibrium, $\beta = \beta'$. Therefore, managers informed about asset i play a linear strategy $x(s_i) = \beta s_i$, where

$$\beta = \frac{(N-\alpha)\sigma_v^2\tau}{\lambda[2(N-\alpha)+(N-2\alpha)(M-1)](\sigma_v^2\tau+1)}. \tag{A.5}$$

An equilibrium is a solution of the system of Eqs. (A.1) and (A.5). This system has a unique solution

$$\beta^*(\alpha) = \frac{\sigma_z\sqrt{(N-\alpha)\tau}}{\sqrt{M(N-\alpha M)(\sigma_v^2\tau+1)}},$$
$$\lambda^*(\alpha) = \frac{\sigma_v^2\sqrt{M(N-\alpha M)(N-\alpha)\tau}}{\sigma_z[2(N-\alpha)+(M-1)(N-2\alpha)]\sqrt{(\sigma_v^2\tau+1)}}. \qquad \square$$

Proof of Proposition 2.

$$\frac{\mathrm{d}\frac{N-\alpha}{N-\alpha M}}{\mathrm{d}\alpha} = \frac{N(M-1)}{(N-\alpha M)^2} > 0.$$

It follows that $\partial\beta^*(\alpha)/\partial\alpha > 0$.

$$\begin{aligned}\mathrm{Var}(\pi_i^*) = {} & \beta^{*2}(1-M\lambda^*\beta^*)^2\big(\mathrm{Var}(v^2)+\sigma_v^2\tau^{-1}\big) + M^2\lambda^{*2}\beta^{*4}\big(\mathrm{Var}(e_i^2)+\sigma_v^2\tau^{-1}\big)\\ & + \lambda^{*2}\beta^{*2}\big(\sigma_v^2+\tau^{-1}\big)\sigma_z^2.\end{aligned}$$

From Proposition 1, I deduce that

$$\beta^*\lambda^* = \frac{(N-\alpha)\sigma_v^2\tau}{(\sigma_v^2\tau+1)[2(N-\alpha)+(M-1)(N-2\alpha)]}.$$

It follows that $\beta^{*2}\lambda^{*2}$ and $\lambda^{*2}\beta^{*4}$ are increasing functions of α. Therefore, a sufficient condition for $\mathrm{Var}(\pi_i^*)$ to be increasing in α is $\mathrm{d}\beta^*(1-M\lambda^*\beta^*)/\mathrm{d}\alpha > 0$.

$$\begin{aligned}&\frac{\mathrm{d}\beta^*(1-M\lambda^*\beta^*)}{\mathrm{d}\alpha}\\ &\quad = \frac{\sigma_z N(M-1)H}{2(\sigma_v^2+\tau^{-1})^{3/2}(N-\alpha M)^{3/2}\sqrt{M(N-\alpha)}[2(N-\alpha)+(M-1)(N-2\alpha)]^2}\end{aligned}$$

where

$$H = \big[2(N-\alpha)+(M-1)(N-2\alpha)\big]^2\tau^{-1}$$

$$+ (N - \alpha)\big[M(2N - 8\alpha + 2\alpha M - 1) + 4\alpha\big].$$

Given that there are $2N_l$ low-quality managers with $N_l \geqslant 1$ and $N_h \geqslant 1$ high-quality managers, $N \geqslant 3$ and $M \geqslant 2$. It follows that $H > 0$ for all $\alpha \in [0, 1]$. Therefore, $\beta^*(1 - M\lambda^*\beta^*)$ is increasing in α and so is $\mathrm{Var}(\pi_i^*)$. □

Proof of Lemma 3. If $N_h^* < \overline{N}_h$, it implies that $F_h(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) = R$. But $F_l(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) < F_h(N_h^*, N_{l1}^*, N_{l2}^*, \alpha)$. As a consequence $F_l(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) < R$. So $N_{l1}^* > 0$, cannot be an equilibrium number of low-quality funds from G_{l1} entering the industry.

Since for any triplet (N_h, N_{l1}, N_{l2}), $F_l(N_h, N_{l1}, N_{l2}, \alpha) < F_h(N_h, N_{l1}, N_{l2}, \alpha)$, the equalities $F_l(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) = R$ and $F_h(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) = R$ cannot hold simultaneously. Hence, there are two types of equilibria:

- Type 1: $F_l(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) = R$ and $F_h(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) > R$.
  In such a case, $N_h^* = \underline{N}_h$ and $N_{li}^* \geqslant 0$.
- Type 2: $F_l(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) < R$ and $F_h(N_h^*, N_{l1}^*, N_{l2}^*, \alpha) \geqslant R$.
  In such a case, $N_h^* \leqslant \underline{N}_h$ and $N_{li}^* = 0$. □

Proof of Lemma 4. Assume that N_{l1} is a continuous variable. Then,

$$\frac{\partial F_l}{\partial N_{l1}}(N_h, N_{l1}, N_{l2}) = \frac{-\alpha[N_{l2}\Pi(N_h + N_{l1}, \alpha) + \Pi(N_h + N_{l2}, \alpha)]}{2(N_h + N_{l1} + N_{l2})^2} + \frac{(N_h + N_{l1} + N_{l2})[(1-\alpha)(N_h + N_{l1}) + N_{l2}]\frac{\partial \Pi}{\partial N_{l1}}}{2(N_h + N_{l1} + N_{l2})^2}.$$

A sufficient condition for $\frac{\partial F_l}{\partial N_{l1}}(N_h, N_{l1}, N_{l2})$ to be negative is then that $\frac{\partial \Pi}{\partial N_{l1}}(N_h + N_{l1}, \alpha) < 0$. This is always true: as the number of competitors increase in a market, equilibrium profit decrease.

Assume that N_{l2} is a continuous variable. Then,

$$\frac{\partial F_l}{\partial N_{l2}}(N_h, N_{l1}, N_{l2}) = \frac{\alpha[\Pi(N_h + N_{l1}, \alpha) - N_{l1}\Pi(N_h + N_{l2}, \alpha) - (N_h + N_{l2})(N_h + N_{l1} + N_{l2})\frac{\partial \Pi}{\partial N_{l2}}]}{(N_h + N_{l1} + N_{l2})^2}.$$

I need to show that $\frac{\partial F_l}{\partial N_{l2}}(N_h, N_{l1}, N_{l2}) > 0$. A sufficient condition is that

$$N_{l1}\Pi(N_h + N_{l2}, \alpha) + (N_h + N_{l2})(N_h + N_{l1} + N_{l2})\frac{\partial \Pi}{\partial N_{l2}} < 0.$$

Let $W = \sigma_z\sigma_v^2\sqrt{\tau}/\sqrt{\sigma_v^2\tau + 1}$, $Y = M_2(1-\alpha) + N_{l1}$, $Z = M_2 + N_{l1} - \alpha$, and

$$D = 2(M_2 + N_{l1} - \alpha) + (M_2 - 1)(M_2 + N_{l1} - 2\alpha).$$

Then $\Pi(M_2, \alpha) = \frac{X\sqrt{YZ}}{D\sqrt{M_2}}$ and

$$N_{l1}\Pi(N_h + N_{l2}, \alpha) + (N_h + N_{l2})(N_h + N_{l1} + N_{l2})\frac{\partial \Pi}{\partial N_{l2}}$$

$$= \Pi(N_h + N_{l2}, \alpha)\Big[- N_{l1} - \tfrac{(N_h+N_{l1}+N_{l2})}{2YZD}\big\{D\big[M_2\big((1-\alpha)Z+Y\big) - YZ\big] \\ - 2M_2YZ(2M_2 + N_{l1} + 1 - 2\alpha)\big\}\Big].$$

So I need

$$(M_2 + N_{l1})\big\{D\big[M_2\big((1-\alpha)Z+Y\big) - YZ\big] \\ - 2M_2YZ(2M_2 + N_{l1} + 1 - 2\alpha)\big\} + 2YZDN_{l1} < 0.$$

This last inequality can be rewritten as

$$YZ\big[D(M_2 - N_{l1}) - 2M_2(2M_2 + N_{l1} + 1 - 2\alpha)(M_2 + N_{l1})\big] \\ + M_2D\big[(1-\alpha)Z + Y\big] < 0, \tag{A.6}$$

$$D(M_2 - N_{l1}) - 2M_2(2M_2 + N_{l1} + 1 - 2\alpha)(M_2 + N_{l1}) \\ < -5M_2^3 - 6M_2^2N_{l1} - MN_{l1}^2 - 3M_2^2(1-2\alpha) - N_{l1}^2 < 0, \\ YZ > M_2^2(1-\alpha) + (2-\alpha)MN_{l1} + N_{l1}^2.$$

I deduce that the LHS of inequality (A.6) is smaller than

$$-5(1-\alpha)M^5 + M^4\big[-(16-11\alpha)N_{l1} + (1-\alpha)(6\alpha-1)\big] \\ + M^3\big[-(13-7\alpha)N_{l1}^2 + (1-12\alpha-6\alpha^2)N_{l1} + (1-\alpha)(2-5\alpha)\big] \\ + M^2\big[-(8-\alpha)N_{l1}^3 + N_{l1}^2(-2+6\alpha) + N_{l1}\alpha(2-\alpha^2)\big] \\ + M\big[-(2-\alpha)\big(N_{l1}^3 + N_{l1}^2\big) - \alpha(1-\alpha)N_{l1}\big] - N_{l1}^4 < 0.$$

So I have the desired result. □

Proof of Proposition 5. *Proof of* (i). Follows directly from the assumption that $F_h(\overline{N}_h, 0, 0, \alpha) > R$ and Lemma 4. If $F_l(\overline{N}_h, 0, 0, \alpha) > R$, then $N_l^* > 0$ while if $F_l(\overline{N}_h, 0, 0, \alpha) < R$, then $N_l^* = 0$.

Proof of (ii). Let $G(N_h, N_l, \alpha) = F_l(N_h, N_l, N_l, \alpha)$ and assume that N_l is a continuous variable. Then N_l^* is the solution of $G(N_h, N_l, \alpha) = R$.

$$\frac{\partial G}{\partial N_l} = \frac{\partial \Pi}{\partial N_l}\left(1 - \frac{2\alpha(N_h + N_l)}{N_h + 2N_l}\right) + \frac{2\alpha N_h \Pi}{(N_h + 2N_l)^2} \tag{A.7}$$

and

$$\frac{\partial \Pi}{\partial N_l} = \frac{X\Pi}{DXY(N_h + N_l)}\left[\frac{(N_h + N_l)D}{2}\big((2-\alpha)Z + 2Y\big) \\ - YZ\big[3 - 2\alpha + 2(N_h + N_l)\big](N_h + Nl)\right] \tag{A.8}$$

with $W = \sigma_z\sigma_v^2\sqrt{\tau}/\sqrt{\sigma_v^2\tau + 1}$, $Y = (1-\alpha)N_h + (2-\alpha)N_l$, $Z = N_h + 2N_l - \alpha$, and $D = 2Z + (N_h + N_l - 1)(Z - \alpha)$.

$$\operatorname*{Lim}_{\alpha \to 0} \tfrac{(N_h+N_l)D}{2}\big((2-\alpha)Z + 2Y\big) - YZ\big[3 - 2\alpha + 2(N_h + N_l)\big](N_h + Nl)$$

$$= (N_h + 2N_l)\{[2(N_h + 2N_l) + (N_h + N_l)(N_h + N_l - 1)](\tfrac{3}{2}N_h + N_L) \\ - (N_h + 2N_l)(N_h + N_l)[3 + 2(N_h + N + l)]\} \\ = -(N_h + 2N_l)(\tfrac{1}{2}N_h^3 + 2N_l^3 + 4N_h^2N_l + \tfrac{13}{2}N_hN_l^2 + \tfrac{7}{2}N_hN_l + 3N_l^2 + \tfrac{3}{2}N_h^2) < 0.$$

Furthermore, $\mathrm{Lim}_{\alpha\to 0}(\partial G/\partial N_l) = \partial\Pi/\partial N_l$. Therefore, by a continuity argument, I derive that there exists $\bar{\alpha} > 0$ such that if $\alpha < \bar{\alpha}$, then $\partial G/\partial N_l < 0$ so that there exists a unique equal-market-size efficient equilibrium.

Proof of (iii).

$$\frac{\mathrm{d}N_l^*}{\mathrm{d}\alpha} = -\frac{\partial G/\partial\alpha}{\partial G/\partial N_l},$$
$$\frac{\partial G}{\partial\alpha} = \frac{\partial\Pi}{\partial\alpha}\left(1 - \frac{2\alpha(N_h + N_l)}{N_h + 2N_l}\right) - \frac{2\Pi}{N_h + 2N_l}.$$

In equilibrium, It is always the case that $(1 - 2\alpha(N_h + N_l)/(N_h + 2N_l)) > 0$,[13] and I know from Proposition 2 that β is increasing in α which implies that $\partial\Pi/\partial\alpha < 0$. Hence, $\partial G/\partial\alpha < 0$.

Since $\partial G/\partial N_l < 0$ and $\partial G/\partial\alpha < 0$, I obtain $\mathrm{d}N_l^*/\mathrm{d}\alpha < 0$. □

Proof of Proposition 6. If N_l is a continuous variable, then N_l^* is the solution of $F(\overline{N}_h, N_l, N_l, \alpha) = R$. This is equivalent to

$$\left(1 - \frac{2\alpha(\overline{N}_h + N_l)}{\overline{N}_h + 2N_l}\right)\Pi(\overline{N}_h + N_l, \alpha) = Q + R.$$

Hence

$$\overline{\Pi}(\alpha) = \frac{2\overline{N}_h + 2N_l^*(\alpha)}{\overline{N}_h + 2N_l^*(\alpha)} \frac{Q + R}{(1 - \frac{2\alpha(\overline{N}_h + N_l)}{\overline{N}_h + 2N_l})}$$

which can be rewritten as

$$\overline{\Pi}(\alpha) = \frac{2(Q + R)}{\frac{\overline{N}_h + 2N_l^*}{\overline{N}_h + N_l} - 2\alpha},$$

$$\frac{\mathrm{d}(\frac{\overline{N}_h + 2N_l^*}{\overline{N}_h + N_l} - 2\alpha)}{\mathrm{d}\alpha} = \frac{\overline{N}_h}{(\overline{N}_h + N_l)^2}\frac{\mathrm{d}N_l^*}{\mathrm{d}\alpha} - 2.$$

If $\alpha < \bar{\alpha}$, then $\mathrm{d}N_l^*/\mathrm{d}\alpha < 0$. This implies that $\mathrm{d}(\frac{\overline{N}_h + 2N_l^*}{\overline{N}_h + N_l} - 2\alpha)/\mathrm{d}\alpha < 0$. Hence, $\overline{\Pi}(\alpha)$ is an increasing function of α. □

Proof of Proposition 7. Denote $M_i^*(\alpha)$, the number of funds trading in market i at time 1.

$$\mathrm{Var}(\tilde{v}_i^1 \mid p_i^1) = \frac{\sigma_v^2[M_i^{*2}(\alpha)\beta_i^2(\alpha) + \sigma_z^2\tau]}{M_i^{*2}(\alpha)\beta_i^2(\alpha)(\sigma_v^2\tau + 1) + \sigma_z^2\tau}.$$

[13] Otherwise, $G(N_h, N_l, \alpha) < 0$.

$\mathrm{Var}(\tilde{v}_i^1 \mid p_i^1)$ is a decreasing function of $M_i^{*2}(\alpha)\beta_i^2(\alpha)$. Therefore, it is sufficient to show that $M_i^*(\alpha)\beta_i(\alpha)$ is an increasing function of α.

$$\frac{\mathrm{d}M^*(\alpha)\beta(\alpha)}{\mathrm{d}\alpha} = \beta\frac{\mathrm{d}M}{\mathrm{d}\alpha} + M\left(\frac{\partial\beta}{\partial M}\frac{\mathrm{d}M}{\mathrm{d}\alpha} + \frac{\partial\beta}{\partial\alpha}\right),$$
$$\frac{\partial\beta}{\partial\alpha} = \frac{N(M-1)\beta}{2(N-\alpha)(N-\alpha M)},$$

and

$$\frac{\partial\beta}{\partial M} = -\frac{(N-2\alpha M)\beta}{2M(N-\alpha M)},$$

where M stands for $M^*(\alpha)$.

It follows that

$$\frac{\mathrm{d}M\beta}{\mathrm{d}\alpha} = \frac{N\beta}{2(N-\alpha M)}\left(\frac{\mathrm{d}M}{\mathrm{d}\alpha} + \frac{M(M-1)}{N-\alpha}\right).$$

Using the proof of Proposition 5, $\mathrm{d}M\beta/\mathrm{d}\alpha > 0$ is equivalent to

$$-\frac{\partial G/\partial\alpha}{\partial G/\partial N_l} + \frac{(\overline{N}_h + N_l)(\overline{N}_h + N_l - 1)}{\overline{N}_h + 2N_l - \alpha} > 0$$

Hence, given that $\partial G/\partial N_l < 0$ on $[0, \bar{\alpha}]$, I need to show that

$$-\left(\overline{N}_h + 2N_l - \alpha\right)\frac{\partial G}{\partial\alpha} + \left(\overline{N}_h + N_l\right)\left(\overline{N}_h + N_l - 1\right)\frac{\partial G}{\partial N_l} < 0. \tag{A.9}$$

Using Eqs. (A.7) and (A.8), I obtain that

$$\lim_{\alpha\to 0} -\left(\overline{N}_h + 2N_l - \alpha\right)\frac{\partial G}{\partial\alpha} + \left(\overline{N}_h + N_l\right)\left(\overline{N}_h + N_l - 1\right)\frac{\partial G}{\partial N_l} < 0.$$

So, by continuity, there exists $\hat{\alpha} \leqslant \bar{\alpha}$ such that $\mathrm{Var}(\tilde{v}_i \mid p_i)$ is decreasing in α on $[0, \hat{\alpha}]$. $\square$

Appendix B. Risk-averse fund managers

Assume that fund manager n ($n = 1, \ldots, N$) is a risk-averse agent with negative exponential utility function who aims at maximizing the expected utility from compensation, i.e., $U(C_n) = -\exp(-\gamma C_n)$.

Given his information, expected utility maximization is equivalent to maximizing

$$U_m(C_n) = E(C_n \mid I_n) - \frac{\gamma}{2}\mathrm{Var}(C_n \mid I_n) \tag{B.1}$$

where $\gamma > 0$ is the coefficient of absolute risk aversion.

First, I derive results on the existence of an equilibrium and trading strategies.

Proposition B.1. (i) *There exists a unique symmetric linear equilibrium.*

(ii) *Trading aggressiveness is increasing in α, i.e., $\partial\beta^*(\alpha)/\partial\alpha > 0$.*

Proof. *Proof of* (i). First, note that as in Proposition 1, uninformed managers do not trade asset since they expect negative profits from trading. Now, assume that the market maker plays a linear strategy and all informed managers $j \neq i$ play a strategy $x(s) = \beta s$. Let

$$X = \frac{N_h - \alpha}{N} x - \frac{(M-1)\alpha}{N} \beta s.$$

Then, the expected utility of compensation manager i derives from buying a quantity x is:

$$\{E(\tilde{v} \mid s) - \lambda[(M-1)\beta s + x]\} X - \frac{\gamma}{2}(1+k)\big(\mathrm{Var}(\tilde{v} \mid s) + \lambda^2 \sigma_z^2\big) X^2. \tag{B.2}$$

We derive that the FOC of expected utility maximization is

$$x = \left\{ (N-\alpha)\frac{\sigma_v^2 \tau}{\sigma_v^2 \tau + 1} + \beta\left[\gamma(1+k)\left(\frac{\sigma_v^2}{\sigma_v^2\tau + 1} + \lambda^2\sigma_z^2\right)\frac{(N-\alpha)(M-1)\alpha}{N} - \lambda(M-1)(N-2\alpha)\right]\right\} s \Bigg/ \left\{ 2\left[\lambda(N-\alpha) + \gamma(1+k)\frac{(N-\alpha)^2}{N}\left(\frac{\sigma_v^2}{\sigma_v^2\tau + 1} + \lambda^2\sigma_z^2\right)\right]\right\}. \tag{B.3}$$

Hence, manager n plays a linear strategy. It follows that in a symmetric linear RPE, informed managers play a strategy $x(s) = \beta s$, where

$$\beta = (N-\alpha)\sigma_v^2\tau \Bigg/ \left\{ \lambda\big(\sigma_v^2\tau + 1\big)\big[2(N-\alpha) + (M-1)(N-2\alpha)\big] + \gamma(1+k)\frac{(N-\alpha)(N-M\alpha)}{N}\big[\sigma_v^2 + \big(\sigma_v^2\tau + 1\big)\lambda^2\sigma_z^2\big]\right\}. \tag{B.4}$$

It is straightforward that Eq. (A.1) still holds. Hence, an equilibrium is a solution of the system of Eqs. (A.1) and (B.4). Plugging (A.1) into (B.4) and rearranging, one obtains

$$\frac{\tau}{\beta} = \frac{M\beta(\sigma_v^2\tau + 1)\tau}{[M^2\beta^2(\sigma_v^2\tau + 1) + \sigma_z^2\tau]}\left[2 + \frac{N-2\alpha}{N-\alpha}(M-1)\right] + \gamma(1+k)\frac{(N-M\alpha)}{N}\left\{1 + \frac{\sigma_z^2 M^2\beta^2\sigma_v^2\tau^2(\sigma_v^2\tau + 1)}{[M^2\beta^2(\sigma_v^2\tau + 1) + \sigma_z^2\tau]^2}\right\}. \tag{B.5}$$

The left-hand side of (B.5) is monotone decreasing in β and goes to infinity as β goes to zero. Let $f(\beta)$ denote the right-hand side as a function of β. $f(0) = \gamma(1+k)\frac{(N-M\alpha)}{N}$. Furthermore, one shows that for all $\beta > 0$, $f'(\beta) > -\tau/\beta^2$. This implies that (B.5) has a unique solution. Hence, there exists a unique equilibrium.

Proof of (ii). The RHS of (B.5) is decreasing in α while the LHS is independent of α. Therefore β^* is increasing in α. □

Proposition B.1 states that risk-aversion does not eliminate risk-taking incentives generated by relative performance objectives. The reason is that the increase in expected compensation generated by higher trading aggressiveness outweights the larger volatility of compensation. Hence, relative performance objectives generate incentives to choose very aggressive strategies even if fund managers are risk-averse.

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